GEOMETRY OF SUBMANIFOLDS OF AN ALMOST CONTACT METRIC MANIFOLD

SHORT SYNOPSIS

Submitted in partial fulfillment of the requirement of the degree of

DOCTOR OF PHILOSOPHY

by

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It is the work that is expected to be carried out under the supervision of Dr. Advin Masih and co-supervision of Dr. Mamta Thakur.

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ABSTRACT

Due to its numerous applications in various branches of mathematics and physics, the study of submanifolds on virtually complex manifolds has attracted considerable attention in differential geometry. This abstract gives a general review of submanifold geometry within the context of “nearly complex manifolds”.

We investigate the elementary characteristics of submanifolds of nearly complex manifolds, concentrating on their inherent geometry. We talk about ideas like immersions, embedding’s, and submanifold types, including those that are holomorphic and completely real. We look into the tangent spaces, normal bundles, and curvature of these submanifolds to learn more about their local and overall characteristics. The study of complex submanifolds, which are submanifolds that are invariant under almost complex structures, is another area we explore. In this article, we'll look at the idea of holomorphicity on complex submanifolds and how it relates to the requirement for integrability of the nearly complex structure.

“The geometry of sub-manifolds in Almost Complex Manifolds” constitutes a fascinating area of research with wide-ranging implications. This abstract serves as an introduction to the subject, inviting further exploration and investigation into this captivating field of mathematics.

Key words: submanifolds, almost complex manifolds, geometry, immersions, tangent spaces.

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1. INTRODUCTION

A thriving topic of differential geometry study is “the geometry of sub-manifolds in nearly Complex Manifolds”. It entails the investigation of lower-dimensional manifolds contained within more complicated higher-dimensional manifolds. Applications of this branch of geometry may be found in many fields, such as algebraic geometry, symplectic geometry, and theoretical physics.

“An Almost Complex Structure on a smooth manifold acts like the complex structure on the complex Euclidean space, is referred to as an Almost Complex Manifold.”

In contrast, submanifolds are lower-dimensional manifolds that are encased within a higher-dimensional manifold. The interaction between the surrounding almost complex structure and the inherent qualities of the submanifold makes the geometry of almost complex manifolds extremely fascinating.

Immersions and embeddings are two essential ideas in the geometry of submanifolds of nearly complex manifolds. An immersion is a smooth map that maintains the differential structure, whereas an embedding is an immersion that is also a topological embedding. The study of submanifolds in nearly complex manifolds includes examining the characteristics of these immersions and embeddings and how they relate to the almost complex structure.

The structure of nearly complex manifolds naturally gives birth to a variety of submanifold types. Submanifolds that have an almost complex structure that completely preserves the tangent spaces are considered to be totally real. Through the concept of holomorphicity, these submanifolds connect to complex geometry and play a key role in symplectic geometry.

Submanifolds that are invariant under the nearly complex structure are called holomorphic submanifolds. They have uses in complex analysis and algebraic geometry and display a variety of geometric features.

Investigating the local and global features of submanifolds of nearly complex manifolds is part of the study of these structures. This entails examining the submanifolds curvature, normal bundles, and tangent spaces. Additionally, holomorphic submanifolds must be characterized and their behavior must be studied, and this requires an understanding of the integrability requirement of the almost complicated structure.

The geometry of Sub-manifolds in Complex Manifolds is an exciting area of exploration that combines symplectic , complex geometry, and differential geometry. The interaction between the

inherent qualities of submanifolds and the geometric forms imposed by the essentially complicated structure is quite rich. Wide-ranging implications of submanifold theory in almost complex manifolds are still being actively investigated in a number of mathematical and scientific fields.

2. LITERATURE REVIEW

2a. SOME DEFINITIONS

1. MANIFOLDS [4]

A Hausdroff space with a differentiable structure is called a manifold. A Topological Space endowed with the characteristics that each of its point has a neighborhood homeomorphic to an open subset of n-dimensional Euclidean Space is known as n-Manifold (or n- dimensional Manifold).Lines and circles falls in category of 1-dimensional manifolds, surfaces are 2-dimensional manifolds.

2. ALMOST COMPLEX MANIFOLDS [4]

A smooth manifold having a smooth linear complex structure on each tangent space is a nearly complex manifold. If on a manifold H there exist an almost Complex Structure Φ(a tensor field Φ on H) such that Φ is an endomorphism of the tangent space T_p(H) $\forall p \in H$ and Φ^2 = -I.
Each nearly complex manifold has an even number of dimensions.

3. HERMITIAN MANIFOLD [14]

A Hermitian Metric is a Riemanian Metric $\bar{k}$ on an Almost Complex Manifold satisfying $\bar{k}$ (Φ P, Φ Q) = $\bar{k}$ (P,Q) for P,Q $\Gamma(TM)$. An almost complex manifold with Hermitian Metric is called a Hermitian Manifold.

4. KAEHLER MANIFOLD [14]

A manifold with three communally harmonious structures namely a Complex Structure, a Riemannian Structure, and a Symplectic Structure is called Kaehlerian Manifold. H is termed **Kaehlerian Manifold** if the almost complex structure Φ of H is parallel i.e.
$(\bar{\nabla}_X \Phi) = 0 \; \forall \; X, Y \in \Gamma(TH)$.

5. CONTACT METRIC MANIFOLD [4]

A Contact metric manifold (H, g) is an odd dimensional Semi-Riemannian manifold if there exist a (1,1) tensor field ξ *(*called the characteristic vector field) and its form η satisfying:

g (JP, JQ) = g (P,Q) - ε η (P) η (Q)

g (ξ, ξ) = ε

J^2(P) = -P+ η (P) ξ , J (P, ξ)=ε η (P)

$d\eta$(P,Q)= g (P, JQ) $\forall P, Q \in \Gamma(TM)$

Where ε = ± 1 . Also J ξ =0 , $\eta \circ$ J =0 , η ξ =ε

Then (J, ξ, η , g) is contact metric structure of H.

TYPES OF CONTACT METRIC MANIFOLD

A contact metric manifold for which we have: $\bar{\nabla}_X \xi = 0$, ($\bar{\nabla}_X$J)Y=0 $\forall X, Y \in \Gamma(TM)$ is **Cosymplectic Manifold .[4]**

Contact metric manifold with $\bar{\nabla}_X \xi = -X + \eta(X)\xi$,

($\bar{\nabla}_X$J)Y = - g (JX,Y) ξ +η(Y)JX $\forall X, Y \in \Gamma(TM)$ is **Kenmotsu Manifold.[4]**

Contact metric manifold for which : $\bar{\nabla}_X \xi =$ J(X) ,

($\bar{\nabla}_X$J)Y = - g (X,Y) ξ +$\varepsilon\eta$(Y)X $\forall X, Y \in \Gamma(TM)$ is **Sasakian Manifold.[4]**

Trans Sasakian Manifold [4] An n-dimensional almost contact metric manifold $\overline{K}$ with almost contact metric structure (μ, ξ, η, g).

μ^2 = -1 + $\eta \otimes \xi$, $\mu \circ \xi$ = 0, $\eta \circ \mu$= 0 , $\eta(\xi)$=1

g(μ O, μ F) = g(O, F) - η(O) η (F) for O, F on $\overline{K}$.

An almost contact structure (μ, ξ, η, g) on $\overline{K}$ is termed Trans-Sasakian if for α and β on $\overline{K}$

($\bar{\nabla}_O \mu$)F= α(g(O, F) ξ - η (F)O) + β{g(μ O, F) ξ - η (F) μ O}

$\bar{\nabla}_O$ ξ = − α μ O + ß(O - η (O) ξ.

5. SUBMANIFOLDS [4]

A submanifold $\overline{K}$ of $\overline{H}$ is a space which itself has the properties of manifold and for which $\dim\overline{K} \geq \dim\overline{H}$ under the inclusion map.

TYPES OF SUBMANIFOLDS

Invariant Submanifold & Anti-Invariant Submanifold [14]

An almost contact metric sub-manifold K of N with almost complex structure Φ and Tq(K) and $Tq(K^{\perp})$the tangent space and the normal space of K at $q \in K$ respectively then

- K is called an **invariant** (or holomorphic) submanifold of N if Tq(K) is invariant i.e. $JTq(K) = Tq(K)$; for each $q \in K$
- K is called an **anti-invariant** (or totally real) submanifold of N if $JTq(K) \subset Tq(K^{\perp})$for each $q \in K$.

CR-Submanifold [14]

A Cauchy-Riemann submanifold or CR-Submanifold is a differentiable submanifold N with a complex distribution: $TN = D \oplus D^{\perp}$

Such that $JD \subset TN$, $J D^{\perp} \subset TN^{\perp}$.

Slant Submanifold [13]

A sub-manifold M of an almost Hermitian manifold (K,J,g) is called a Slant sub-manifold if angle $\theta(Y)$ is unaffected by the chosen unit vector $Y \in T_p M$;$p \in M$ and θ being the slant angle $[0 \leq \theta \leq \frac{\pi}{2}]$. If θ=0 then slant sub-manifold is generalisation of Complex(invariant) manifold and if $\theta = \pi/2$ then the slant submanifold is generalisation of totally real(anti-invariant)manifold.

Semi slant Submanifold [13]

A distribution L on a sub-manifold M of an almost hermitian-manifold M is a slant distribution if the angle between JU and Lx is constant(for each $U \in Lx$,), i.e., independent of $x \in M$ and $U \in Lx$. A sub-manifold M of $\overline{M}$ is slant sub-manifold if the tangent bundle TM of M is slant.

A semi-slant sub-manifold M of $\overline{M}$ admits two orthogonal complementary distributions L_1 and L_2 where L_1 is holomorphic i.e., $JL_1 = L_1$ and L_2 is slant with $\theta \neq 0$.

Bi Slant Submanifold [13]

A Bi Slant submanifold K of an almost contact metric manifold N if for each point $k \in K$ it comprises of three orthogonal complementary distributions namely D^*, D_{1^*} and D_{2^*}

(i) $TK = D^* \oplus D_{1^*} \oplus D_{2^*} \oplus \langle \xi' \rangle$

(ii) The distribution D^* is invariant i.e. $J(D^*) = D^*$

(iii) $J(D_{1^*}) \perp D_{2^*}, \phi(D_{2^*}) \perp D_{1^*}$

(iv) The distributions D1*, D2* are slant with slant angles

Let m1, m2 and m3 be the dimensions of D*, D1* and D2* respectively

If $m1 = 0$, $m2 \neq 0$, $m3 \neq 0$ and $0 < \theta1 < \pi/2$, $0 < \theta2 < \pi/2$, then K is a bi-slant sub-manifold.

Quasi Bi Slant Submanifold and Quasi Hemi Slant Submanifold [13]

A sub-manifold K of an almost contact metric manifold N is Quasi Bi slant sub-manifold if it admits three orthogonal complementary distributions $-D'$, $D_{1'}$ and $D_{2'}$ at point $k \in K$ such that

(i) $TK = D' \oplus D_{1'} \oplus D_{2'} \oplus \langle \xi' \rangle$

(ii) $\phi(D') = D'$ i.e. D' is invariant

(iii) $\phi(D_{1'}) \perp D_{2'}, \phi(D_{2'}) \perp D_{1'}$

(iv) D1', D2' are slant with angles $\theta1$, $\theta2$

Let m1, m2 and m3 be the dimensions of D', D1', D2' and D3' respectively

- If $m1 \neq 0$, $m2 \neq 0$, $m3 \neq 0$, $0 < \theta1 < \pi/2$ and $\theta2 = \pi/2$, then K is a **Quasi Hemi Slant submanifold.**
- If $m1 \neq 0$, $m2 \neq 0$, $m3 \neq 0$ and $\theta1, \theta2 \neq 0, \pi/2$, then K is **Proper Quasi Bi-Slant submanifold.**

2b. CR submanifolds of Kaehler Manifolds

Meaning

A CR submanifold of a complex-manifold is the one equipped with a compatible CR-structure.

A CR-Structure on a manifold defines a distribution that is almost complex and determines how the submanifold interacts with complex tangent directions."

Let $\bar{\mathbf{K}}$ Kaehler manifold, a submanifold N of $\bar{\mathbf{K}}$ is known as CR submanifold with differentiable distribution L : $\mathbf{p} \rightarrow \mathbf{L_p} \subseteq \mathbf{T_pN}$ such that :

(a) $\mathbf{JL_p} = \mathbf{L_p}$ for each p ϵ N , i.e., L is holomorphic

(b) the complementary orthogonal distribution $L^{\perp}$: $\mathbf{p} \rightarrow \mathbf{L_p}^{\perp} \subseteq \mathbf{T_pN}$ satisfies
$\mathbf{JL_p}^{\perp} \subseteq \mathbf{T_p}^{\perp}\mathbf{N}$ for each $\mathbf{p} \epsilon \mathbf{N}$ is totally real.

CR- SUBMANIFOLDS OF A KAEHLER MANIFOLD- I (A. BEJANCU) [1]

An m-dimensional Riemannian submanifold, M, immersed in Kaehler manifold $\bar{K}$ ($\dim\bar{K} = n$) and g (resp. go) represents the Kaehlerian Metric on $\bar{K}$ (Riemannian Metric on M resp.), ϕ represents the almost complex structure on $\bar{K}$, ϕ' denotes isometric immersion of M into $\bar{K}$.
Consider L : $x \rightarrow L_x \subseteq T_xM$ be a differential distribution on M. Assuming, L is consistent with J, i.e., ϕ (Lx) = Lx ($\forall$ x ϵ *M*),.
$L^{\perp}$: $x \rightarrow L_x^{\perp} \subseteq T_x M$ is the complementary orthogonal distribution.

Let *R* (resp. *Q)* be the projections defined on the distribution *L* (resp. $L^{\perp}$*) such that*

$$R^2 = R,\ S^2 = S, \qquad RS = SR = 0,\ g \bullet (R\ X\ S) = 0$$

If dim $L_x^{\perp}$ =0 (dim L_x =0 resp.) for some x ϵ *M* , then M, the *CR* submanifold is a Complex Submanifold (resp. totally real submanifold) of $\bar{K}$.

Consider
$\phi\xi = A''\xi + B'\xi + D'\xi$;where $\xi \boldsymbol{\epsilon}$ normal bundle
where *A''ξ and B'ξ* is the horizontal component and vertical component of $\phi\xi$ *respectively* and $D'\xi$ *is* the normal portion. So *A''* is horizontal and *B'* is vertical valued I-form on the normal bundle.

lf X ϵ M , ϕSX is a segment in the normal bundle of M,

$$B'\phi SX + SX = 0,$$

$$A''\phi SX = D'\phi SX = 0.$$

Solving $\phi(\phi\xi = A''\xi + B'\xi + D'\xi)$ and solving

$$D'^2\xi + \phi B'\xi + \xi = 0$$

$$\phi A''\xi + A''D'\xi = 0$$

$$B'D'\xi = 0$$

Also $D'^3 + D' = 0$ on the normal bundle .

Considering Gauss and Weingarten equations:

$$\nabla_X Y = \nabla_X Y + h(X, Y),$$
$$\nabla_X \xi = -A_\xi X + \nabla^\perp_X \xi,$$

∇ : Riemannian Connection on *N*,

$\boldsymbol{h}$: 2nd fundamental form.

A: endomorphic mapping of the tangent of *M*

∇ : Kaehlerian Connection

$$R(\nabla_X \phi RY) - R(A''_{\phi SY} X) = \phi R(\nabla_X Y) + A''h(X,Y)$$

$$S(\nabla_X \phi RY) - S(A''_{\phi SY} X) = B'h(X,Y)$$

$$h(X, \phi RY) + \nabla^\perp_X \phi SY = \phi S(\nabla_X Y) + D'h(X,Y)$$

Differentiating $\phi\xi = A''\xi + B'\xi + D'\xi$ and solving for horizontal, vertical and normal parts

$$R(\nabla_X A''\xi) + R(\nabla_X B'\xi) + \phi R(A''_\xi X) = R(A''_{D'\xi} X) + A''(\nabla^\perp_X \xi)$$

$$S(\nabla_X A''\xi) + S(\nabla_X B'\xi) = S(A''_{D'\xi} X) + B'(\nabla^\perp_X \xi)$$

$$h(X, A''\xi) + h(X, B'\xi) + \nabla^\perp_X D'\xi + \phi(SA''_\xi X) = D'(\nabla^\perp_X \xi)$$

$\forall$X on M and normal ξ.

Definition

"On M,L is called parallel if ∇_U V ϵ L for U, V ϵ L.

* L is **involutive**, iff, h(U, ϕ V) = h(ϕ U, V) $\forall$ U, V ϵ L.

* L is involutive if it is parallel with respect to U"

Definition

The CR-sub-manifold M is called L-Totally Geodesic (resp. $L^{\perp}$ Totally Geodesic) if h(U,V) = 0 for each U, V ϵ L (resp. U, V ϵ $L^{\perp}$)."

CR-SUBMANIFOLDS OF A KAEHLERMANIFOLD- I (B.Y. CHEN) [12]

$\bar{\mathbf{M}}$ denote a Kaehler Manifold then K is called a CR submanifold of $\bar{\mathbf{M}}$ let there be a differentiable distribution on K namely L and distribution $L^{\perp}$(orthogonal complement of L) such that ϕ (Lx) = Lx and $\phi \mathbf{L_x}^{\perp} \subseteq \mathbf{T_x}^{\perp}\mathbf{K}$

ϕ be the complex structure on K. Denote by < , > the metric of $\overline{\mathbf{M}}$ and N. ∇ and $\bar{\nabla}$ denote covariant derivatives on K and $\overline{\mathbf{M}}$ respectively.

$$\bar{\nabla}_X \mathrm{Y} = \nabla_X Y + h(X,Y)$$

$$\bar{\nabla}_X \xi = -\mathrm{F}_{\xi} X + \nabla^{\perp}_X \xi$$

Are the Gauss and Weingarten equations; X, Y are tangents to K ; $\boldsymbol{\xi}$ is normal to K,

h : 2nd fundamental form, and $\nabla^{\perp}_X$: normalconnection in $\mathbf{T}^{\perp}\mathbf{K}$. $\mathbf{F_{\xi}}$ and h are related as

$$< \mathrm{F}_{\xi} X, Y > = < h(X,Y), \xi >$$

X tangent to K, $\phi X = RX + SX$; RX is tangential , SX is normal part of ϕ X.

$\boldsymbol{\xi}$ normal to K, $\phi\boldsymbol{\xi} = \boldsymbol{e\xi} + \boldsymbol{f\xi}$; $e\boldsymbol{\xi}$ is tangential and f $\boldsymbol{\xi}$ is normal portion of $\phi\boldsymbol{\xi}$.

Lemma 1. Let $\bar{\mathbf{M}}$ be a Kaehler Manifold and K be a CR-submanifold of $\bar{\mathbf{M}}$. Then

$$< \nabla_U Z, X > = < \phi \mathrm{F}_{\phi Z} U, X >$$

$$F_{\phi Z}W = F_{\phi W}Z$$

$$F_{\phi \xi}X = -F_{\xi}\phi X$$

U is tangent to K and $X \in L$, $Z, W \in L^{\perp}$, and $\boldsymbol{\xi} \in v$.

Lemma 2. Let $\bar{\mathbf{M}}$ be a Kaehler Manifold and K be a CR-submanifold of $\bar{\mathbf{M}}$ Then we have $\mathbf{L_W}\phi\mathbf{Z} - \mathbf{L_Z}\phi\mathbf{W} \in \phi\mathbf{L}^{\perp}$ for Z, $W \in L^{\perp}$.

Lemma 3.. Let $\bar{\mathbf{M}}$ be a Kaehler Manifold ,K be a CR Submanifold of $\bar{\mathbf{M}}$ .The distribution L is integrable iff

$$< \mathbf{h}(\mathbf{X}, \phi\mathbf{Y}), \phi\mathbf{Z} > = < \mathbf{h}(\phi\mathbf{X}, \mathbf{Y}), \phi\mathbf{Z} >$$

For $X, Y \in L$, and $Z \in L^{\perp}$.

Lemma 4. The leaf $K^{\perp}$ of $L^{\perp}$ is totally geodesic in Kaehler Manifold K iff

$$< \mathbf{h}(\mathbf{L}, \mathbf{L}^{\perp}), \phi\mathbf{L}^{\perp} > = \mathbf{0}$$

Lemma 5. If L is integrable and $< \mathbf{h}(\mathbf{L}, \mathbf{L}^{\perp}), \phi\mathbf{L}^{\perp} > = \mathbf{0}$ holds then for any $X \in L$ and ξ in$\in$ $\mathbf{L}^{\perp}$ **,**then

$$\mathbf{F_\xi}\phi\mathbf{X} = -\phi\mathbf{F_\xi X}.$$

3. DESCRIPTION OF BROAD AREA

DIIFERENTIAL GEOMETRY

The mathematical field of differential geometry examines the geometry of smooth manifolds, often known as smooth spaces and smooth forms. It employs the methods of multilinear algebra, differential algebra, and integral calculus. The study of spherical geometry dates back to antiquity and is the field's first source of inspiration. It also has connections to astronomy, Earth geodesy, and Lobachevsky's subsequent research on hyperbolic geometry. The study of these forms in the 18th and 19th centuries laid the developmental ground of modern day differential geometry.

In mathematics, manifolds are essential concepts with a long history. They have uses in physics, computer science, and engineering, among other disciplines. In the 17th century, the idea of manifolds first emerged. The concept of manifolds, however, begins to develop in the 19th century.

The study of properties of things that are preserved through continuous deformations like bending and stretching is known as topology. Topology was initially developed in the early 20th century. From a geometric standpoint, topologists started to investigate manifolds, concentrating on their inherent qualities like connectedness, compactness, and orientability.

A manifold is essentially a collection of points with a neighborhood surrounding each one that is homomorphic to Euclidean space. Thus, the manifold appears flat locally, like a segment of a plane or a curved surface. Manifolds can have any finite number of dimensions. An n dimensional manifold is called n-manifold. A curve is the simplest example of a one-dimensional manifold.

Manifolds can be further classified based on the smoothness of their structure. A smooth manifold is one where the transition functions between charts are infinitely differentiable. Smooth manifolds are the primary focus in differential geometry. At each point on a smooth manifold, there exists a tangent space. The tangent space is a vector-space isomorphic to Euclidean space and the tangent bundle is collection of all tangent spaces of the manifold.

4. OBJECTIVES OF THE STUDY

1. The goal is to classify various submanifold types of contact metric manifolds. In order to do this, it is necessary to recognize and investigate numerous classes of submanifolds, including fully umbilical submanifolds, entirely geodesic submanifolds, and specific submanifold varieties like Sasakian or cosymplectic submanifolds.
2. The study of differential equations and geometric analysis can both be enhanced by studying submanifolds on nearly contact metric manifolds.
3. Examining the connections between submanifolds and other geometric structures in nearly contact metric manifolds is a frequent task. Investigating connections to symplectic geometry, Kähler geometry, or contact geometry can provide deeper insights for understanding submanifold geometry.

5. METHODOLOGY

Research methodology adopted

1. Investigate the geometry submanifolds of even and odd dimensional manifolds and give their examples.

2. Test the existence of submanifold over the ambient manifold.

3. Investigate the geometry of submanifolds of the ambient manifold by establishing the results on integrabilities of distributions and totally geodesicness of distribution and totally umbilical property etc.

6. EXPECTED OUTCOME OF THE RESEARCH

The goal of this proposed work is to establish results on

1. Quasi Bi slant submanifolds, quasi-Hemi slant sub-manifolds over Semi Riemanian manifold.
2. Pointwise Warped Product submanifolds over Riemanian manifolds and Semi-Riemannian Manifolds
3. Quasi Hemi slant Warped Product submanifolds over Semi Riemanian manifold.

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